CBSE 2025 EXAMINATIONS
XII MATHEMATICS (041)
Series YXW2Z ♦ Q.P. Code 65/2/1; 65/2/2; 65/2/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/2/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(a) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (d) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

Sol. (a) Projection vector of vector $\vec{a}$ on vector $\vec{b}$ is $(\vec{a}.\hat{b})\hat{b} = \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

02. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

(a) $(0, 2)$ (b) $(-\infty, 2]$ (c) $[1, 2]$ (d) $[2, \infty)$

Sol. (d) $f'(x) = 2x - 4 \geq 0$ in $x \in [2, \infty)$. So, function is increasing in the interval $[2, \infty)$.

03. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is

(a) $\int_0^{2a} f\left(\frac{x}{2}\right)dx$ (b) $\int_0^{a} f(x)\,dx$ (c) $2\int_a^{0} f(x)\,dx$ (d) $2\int_0^{a} f(x)\,dx$

Sol. (d) $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$, as $f(2a - x) = f(x)$.

04. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is

(a) -8 (b) 0 (c) 6 (d) 8

Sol. (d) As the matrix A is symmetric so, $a_{ij} = a_{ji}$.

Now $a_{21} = a_{12} \Rightarrow 6x = 12 \Rightarrow x = 2$;

Also $a_{31} = a_{13} \Rightarrow 8x = 4y \Rightarrow y = 2x = 4$.

Hence, $(2x + y) = 2x + y = 4 + 4 = 8$.

05. If $y = \sin^{-1} x$, $-1 \le x \le 0$, then the range of y is

(a) $\left(-\frac{\pi}{2}, 0\right)$ (b) $\left[-\frac{\pi}{2}, 0\right]$ (c) $\left[-\frac{\pi}{2}, 0\right)$ (d) $\left(-\frac{\pi}{2}, 0\right]$

Sol. (b) As $-1 \le x \le 0 \Rightarrow \sin^{-1}(-1) \le \sin^{-1} x \le \sin^{-1} 0$

$\Rightarrow -\frac{\pi}{2} \le y \le 0$

So, the range is $y \in \left[-\frac{\pi}{2}, 0\right]$.

06. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive direction of x, y and z-axis respectively, then θ is

(a) $-\frac{\pi}{3}$ only (b) $\frac{\pi}{3}$ only (c) $\frac{\pi}{6}$ (d) $\pm\frac{\pi}{3}$

Sol. (b) Using $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we get $\cos^2\frac{3\pi}{4} + \cos^2\frac{\pi}{3} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \left(-\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2}$ $\therefore \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$.

Remark None of the options are correct.

07. If E and F are two events such that $P(E) > 0$ and $P(F) \ne 1$, then $P(\bar{E} \mid \bar{F})$ is

(a) $\frac{P(\bar{E})}{P(\bar{F})}$ (b) $1 - P(\bar{E} \mid F)$ (c) $1 - P(E \mid F)$ (d) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Sol. (d) $P(\bar{E} \mid \bar{F}) = \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})} = \frac{P(\overline{E \cup F})}{P(\bar{F})} = \frac{1 - P(E \cup F)}{P(\bar{F})}$.

08. Which of the following can be both a symmetric and skew-symmetric matrix?

(a) Unit Matrix (b) Diagonal Matrix (c) Null Matrix (d) Row Matrix

Sol. (c) Null Matrix (if it is a square matrix).

09. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is

(a) $x = 4t + 3,\ y = -3t + 1,\ z = 7t + 2$ (b) $x = 3t + 4,\ y = t + 3,\ z = 2t + 7$

(c) $x = 3t + 4,\ y = t - 3,\ z = 2t + 7$ (d) $x = 3t + 4,\ y = -t + 3,\ z = 2t + 7$

Sol. (c) Using $x = x_1 + a\lambda,\ y = y_1 + b\lambda,\ z = z_1 + c\lambda$, we get $x = 3t + 4,\ y = t - 3,\ z = 2t + 7$.

10. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2. It is known that $A \ne B \ne I$ and $A^{-1} \ne B$.

Their answers are given as

Abhay : $6AB$

Bina : $7AB - BA$

Chhaya : $8AB$

Devesh : $7BA - AB$

Who answered it correctly?

(a) Abhay (b) Bina (c) Chhaya (d) Devesh

Sol. (b) $4AB+3(AB+BA)-4BA=4AB+3AB+3BA-4BA=7AB-BA$.

11. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is

(a) 0.1 cm/s (b) 0.5 cm/s (c) 1 cm/s (d) 1.1 cm/s

Sol. (c) Volume of the cylindrical tank, $V=\pi r^2 h=100\pi\times h$ [$\because r=10$ cm

$$\Rightarrow \frac{dV}{dt}=100\pi\times\frac{dh}{dt}$$

$$\Rightarrow 100\pi=100\pi\times\frac{dh}{dt}$$

$$\therefore \frac{dh}{dt}=1 \text{ cm/s}.$$

12. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p}+\vec{q})$ will be a unit vector for what value of α?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Sol. (d) According to question, $|\vec{p}+\vec{q}|=1 \Rightarrow |\vec{p}+\vec{q}|^2=1^2$

$$\Rightarrow (\vec{p}+\vec{q}).(\vec{p}+\vec{q})=1$$

$$\Rightarrow |\vec{p}|^2+|\vec{q}|^2+2\vec{p}.\vec{q}=1$$

$$\Rightarrow 1^2+1^2+2|\vec{p}|.|\vec{q}|\cos\alpha=1 \Rightarrow 2\times1\times1\cos\alpha=-1$$

$$\Rightarrow \cos\alpha=-\frac{1}{2} \quad \therefore \alpha=\frac{2\pi}{3}.$$

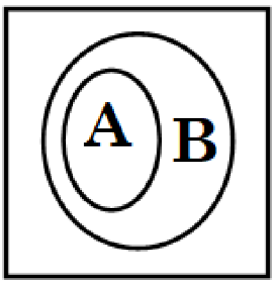
13. The line $x=1+5\mu$, $y=-5+\mu$, $z=-6-3\mu$ passes through which of the following point?

(a) (1, –5, 6) (b) (1, 5, 6) (c) (1, –5, –6) (d) (–1, –5, 6)

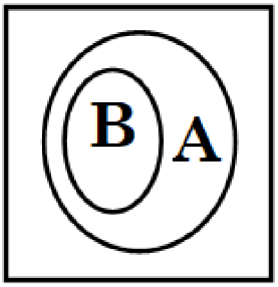
Sol. (c) For $\mu=0$, we get (1, –5, –6). Hence, the point (1, –5, –6) satisfies the given line.

14. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B?

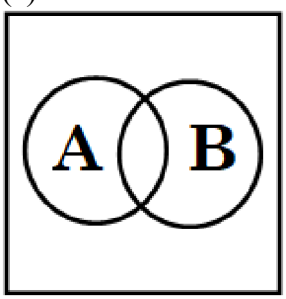
(a)



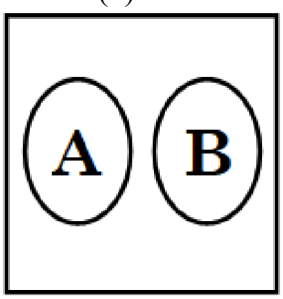
(b)



(c)

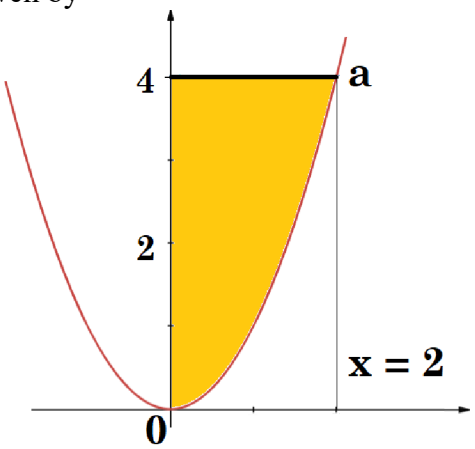


(d)



Sol. (b) Note that, every differentiable function is continuous, but not every continuous function is differentiable.
This means that the set of differentiable functions (B) is a subset of the set of continuous functions (A). Mathematically, $B \subseteq A$.

15. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by



(a) $\int_0^2 x^2\,dx$

(b) $\int_0^2 \sqrt{y}\,dy$

(c) $\int_0^4 x^2\,dx$

(d) $\int_0^4 \sqrt{y}\,dy$

Sol. (d) Shaded area $= \int_0^4 \sqrt{y}\,dy$.

16. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?
(a) The objective function maximizes the difference of the profit earned from products X and Y.
(b) The objective function measures the total production of products X and Y.
(c) The objective function maximizes the combined profit earned from selling X and Y.
(d) The objective function ensures the company produces more of product X than product Y.

Sol. (c) The objective function maximizes the combined profit earned from selling X and Y.

17. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?
(a) $A = B$ (b) $AB = BA$ (c) $A = O$ or $B = O$ (d) $A = I$ or $B = I$

Sol. (b) $A^2 - B^2 = (A - B)(A + B)$
$\Rightarrow AA - BB = AA + AB - BA - BB$ [By cancellation laws
$\Rightarrow O = AB - BA$ $\therefore AB = BA$.

18. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is
(a) 0 (b) 1 (c) 2 (d) 3

Sol. (b) Note that, $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$ implies $3\left(\frac{dy}{dx}\right)^2 \times \frac{d^2y}{dx^2} = 0$.
So, order (p) : 2 and degree (q) : 1. Hence, $(p - q) = 2 - 1 = 1$.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** $A = \text{diag}\begin{bmatrix}3 & 5 & 2\end{bmatrix}$ is a scalar matrix of order 3×3.

Reason (R) : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

Sol. (d) Assertion (A) is false but Reason (R) is true.

20. **Assertion (A) :** Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R) : The optimal solution for a Linear Programming Problem exists only at one or more corner point (s) of the feasible region.

Sol. (d) Assertion (A) is false. Also, Reason (R) is true.

Note that, the optimal solution occurs only at the corner point (s) of feasible region.

Remark In my opinion, Reason (R) is false. Recall that the optimal solution (maximum or minimum value) of a linear programming always occurs at the corner points of the feasible region. But if two corner points of the feasible region are both optimal solutions of the same type (i.e., both produce the same maximum or minimum), then any point on the line segment joining these two corner points is also an optimal solution of the same type.

Then why I marked Reason (R) as true? Because in 2024 Annual Exams (QP Code 65/4/1 Q20), similar statement in Reason (R) was marked as true in the Marking Scheme (which otherwise was false; in my opinion - as per explanation given above).

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

Sol. (a) Since $\vec{a}$ makes equal angles with all the three axes so, $\alpha = \beta = \gamma$.

Then using $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we get $3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$.

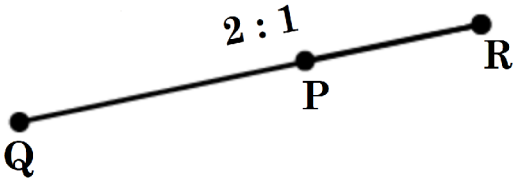
Therefore, the unit vector along $\vec{a}$ will be $\hat{a} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$.

Hence, $\vec{a} = |\vec{a}|\hat{a} = 5\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = 5(\hat{i} + \hat{j} + \hat{k})$.

OR

(b) As $QR = \frac{3}{2}QP$ so, $QP + PR = \frac{3}{2}QP$ i.e., $PR = \frac{1}{2}QP$ i.e., $\frac{QP}{PR} = \frac{2}{1}$. Clearly, P divides QR in $2:1$ internally.

Consider the diagram shown.



Then $\overrightarrow{OP} = \frac{2\overrightarrow{OR} + 1\overrightarrow{OQ}}{2+1}$

$\Rightarrow 3\overrightarrow{OP} = 2\overrightarrow{OR} + 1\overrightarrow{OQ} = 2\overrightarrow{OR} + \vec{\beta}$

$\therefore \overrightarrow{OR} = \frac{3\vec{\alpha} - \vec{\beta}}{2}$. $\left[\because \overrightarrow{OP} = \vec{\alpha}\right.$

Alternatively, let $\overrightarrow{OR} = \vec{r}$.

Now $\overrightarrow{OR} - \overrightarrow{OQ} = \frac{3}{2}\left[\overrightarrow{OP} - \overrightarrow{OQ}\right]$ $\left[\because\ QR = \frac{3}{2}\ QP\right.$

$\Rightarrow \vec{r} - \vec{\beta} = \frac{3}{2}\left[\vec{\alpha} - \vec{\beta}\right]$

$\Rightarrow \vec{r} = \frac{3}{2}\left[\vec{\alpha} - \vec{\beta}\right] + \vec{\beta}$

$\therefore \vec{r} = \frac{3\vec{\alpha} - \vec{\beta}}{2} = \frac{3\vec{\alpha}}{2} - \frac{\vec{\beta}}{2}$.

22. Evaluate $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\, dx$.

Sol. $\sqrt{1+\sin 2x} = \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \sqrt{(\sin x + \cos x)^2} = |(\sin x + \cos x)| = (\sin x + \cos x)$

$\because\ x \in \left[0, \frac{\pi}{4}\right]$, where $\sin x \geq 0, \cos x \geq 0$. Clearly, $(\sin x + \cos x) \geq 0$.

Now $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\, dx = \int_0^{\frac{\pi}{4}} (\sin x + \cos x)\, dx = \left[\sin x - \cos x\right]_0^{\pi/4}$

$= \left[\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right] - \left[\sin 0 - \cos 0\right] = 0 - (0-1) = 1.$

23. Find the values of 'a' for which $f(x) = \sin x - a x + b$ is increasing on $\mathbb{R}$.

Sol. For $f(x) = \sin x - ax + b$, $f'(x) = \cos x - a$.

For increasing function, $f'(x) \geq 0$ i.e., $\cos x - a \geq 0$

$\Rightarrow \cos x \geq a$

Since $\cos x \in [-1, 1]\ \forall\ x \in \mathbb{R}$. So, $a \leq -1$.

Hence, $a \in (-\infty, -1]$.

24. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

Sol. Since $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear vectors.

So, $\vec{\alpha} = \lambda\vec{\beta}$ i.e., $(x-2)\vec{a} + \vec{b} = \lambda\left[(3+2x)\vec{a} - 2\vec{b}\right]$

On comparing the coefficients of $\vec{a}$ and $\vec{b}$ on both the sides, we get
$(x-2) = (3+2x)\lambda$ and $1 = -2\lambda$.

On solving, we get $\lambda = -\frac{1}{2}$ and $(x-2) = (3+2x)\left(-\frac{1}{2}\right)$

$\Rightarrow 2x - 4 = -3 - 2x$

$\Rightarrow 4x = 1$

$\therefore x = \frac{1}{4}$.

25. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$.

OR

(b) If $f(x)=\begin{cases}2x-3, & -3\le x\le -2\\ x+1, & -2<x\le 0\end{cases}$, check the differentiability of $f(x)$ at $x=-2$.

Sol. (a) We have $x=e^{\frac{x}{y}}$

Take log on both the sides, we get $\log x=\log e^{\frac{x}{y}} \quad \Rightarrow \log x=\frac{x}{y}\log e$

$\Rightarrow \log x=\frac{x}{y} \qquad [\because \log e=1$

$\Rightarrow y\times\log x=x$

On differentiating w.r.t. x, we get $y\times\frac{1}{x}+\log x\times\frac{dy}{dx}=1$

$\Rightarrow \log x\times\frac{dy}{dx}=1-\frac{y}{x}=\frac{x-y}{x}$

$\therefore \frac{dy}{dx}=\frac{x-y}{x\log x}$.

OR

(b) Here $f(-2)=2(-2)-3=-7$.

$Rf'(-2)=\lim\limits_{x\to -2^+}\frac{f(x)-f(-2)}{x-(-2)}=\lim\limits_{x\to -2^+}\frac{(x+1)-(-7)}{x+2}=\lim\limits_{x\to -2^+}\frac{(x+8)}{x+2}=\frac{(-2+8)}{-2+2}=\frac{6}{0}=$ Not defined.

As $Rf'(-2)$ is not defined at $x=-2$. So, $f(x)$ is not differentiable at $x=-2$.

Note that, $Lf'(-2)=\lim\limits_{x\to -2^-}\frac{f(x)-f(-2)}{x-(-2)}=\lim\limits_{x\to -2^-}\frac{(2x-3)-(-7)}{x+2}=\lim\limits_{x\to -2^-}\frac{(2x+4)}{x+2}=\lim\limits_{x\to -2^-}\frac{2(x+2)}{x+2}$

$\Rightarrow Lf'(-2)=\lim\limits_{x\to -2^-}(2)=2$.

We do **not** need to find $Lf'(-2)$ as $Rf'(-2)$ is not defined.

Alternatively, we can show that $f(x)$ is discontinuous at $x=-2$ and hence, it will not be differentiable at $x=-2$.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. (a) Solve the differential equation $2(y+3)-xy\frac{dy}{dx}=0$; given that $y(1)=-2$.

OR

(b) Solve the following differential equation : $(1+x^2)\frac{dy}{dx}+2xy=4x^2$.

Sol. (a) We have $2(y+3)-xy\frac{dy}{dx}=0$ i.e., $2(y+3)=xy\frac{dy}{dx}$

$\Rightarrow \int\frac{2\,dx}{x}=\int\frac{y\,dy}{y+3}$

$\Rightarrow \int\frac{2\,dx}{x}=\int\left[1-\frac{3}{y+3}\right]dy$

$\Rightarrow 2\log|x|=y-3\log|y+3|+C$

As $y(1)=-2$ so, $2\log|1|=-2-3\log|-2+3|+C \quad \Rightarrow C=2 \qquad [\because \log|1|=0$

Hence, the required solution is $2\log|x|=y-3\log|y+3|+2$.

OR

(b) Given $(1+x^2)\dfrac{dy}{dx}+2xy=4x^2$

$\Rightarrow \dfrac{dy}{dx}+\left(\dfrac{2x}{1+x^2}\right)y=\dfrac{4x^2}{(1+x^2)}$

On comparing with $\dfrac{dy}{dx}+P(x)\,y=Q(x)$, we get $P(x)=\left(\dfrac{2x}{1+x^2}\right)$, $Q(x)=\dfrac{4x^2}{(1+x^2)}$

Integration Factor $=e^{\int\left(\frac{2x}{1+x^2}\right)dx}=e^{\log(1+x^2)}=(1+x^2)$.

So, the solution is given by $(1+x^2)\,y=\int\dfrac{4x^2}{(1+x^2)}\times(1+x^2)\,dx+C$

That is, $(1+x^2)\,y=\int 4x^2\,dx+C$

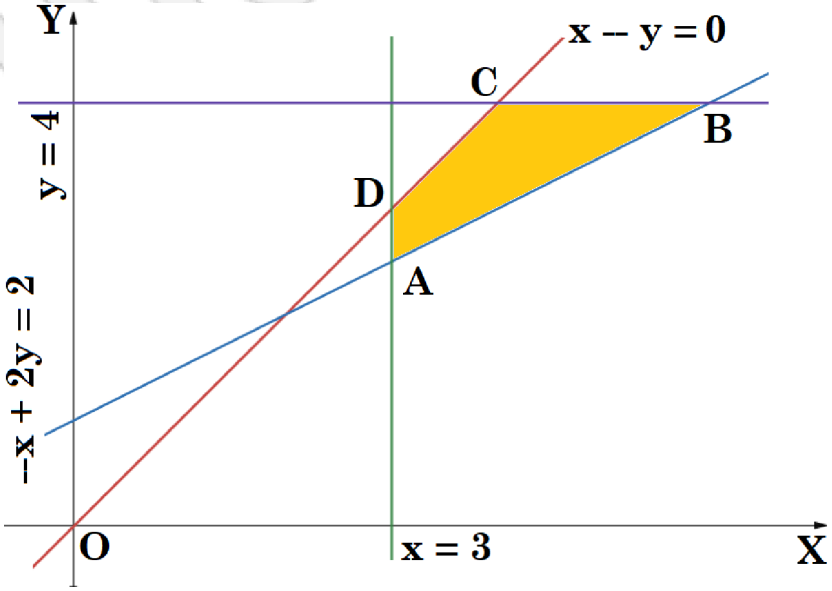
$\therefore\ (1+x^2)\,y=\dfrac{4x^3}{3}+C$ is the required solution of given D.E.

27. Let R be a relation defined over N, where N is set of natural numbers, defined as "$m\,R\,n$ if and only if m is a multiple of n; $m, n\in N$." Find whether R is reflexive, symmetric and transitive or not.

Sol. Since m is always a multiple of m for all $m\in N$. So, R is reflexive.
Also note that $6\,R\,3\neq 3\,R\,6$, as 6 is multiple of 3, but 3 is not a multiple of 6.
So, R is not symmetric.
Let m is a multiple of n and n is a multiple of r.
That is, $m=k\,n$ and $n=p\,r$.
Then, $m=k\times p\,r=(k\,p)\,r$, (for some naturals k and p).
Clearly, m will be a multiple of r also.
That is, $m\,R\,n$ and $n\,R\,r$ implies, $m\,R\,r$.
Hence, R is transitive.

28. Solve the following linear programming problem graphically.
Minimize $Z=x-5y$
Subject to the constraints $x-y\geq 0,\ -x+2y\geq 2,\ x\geq 3,\ y\leq 4,\ y\geq 0$.

Sol. Consider the graph shown below.



The corner points are $A\left(3, \frac{5}{2}\right)$, $B(6, 4)$, $C(4, 4)$ and $D(3, 3)$.

Corner Points	Value of Z
$A\left(3, \frac{5}{2}\right)$	$3-5\times\frac{5}{2}=-\frac{19}{2}$
$B(6, 4)$	$6-5\times4=-14$
$C(4, 4)$	$4-5\times4=-16 \leftarrow$ Minimum
$D(3, 3)$	$3-5\times3=-12$

The minimum value of Z is '–16' and it is obtained at $(4, 4)$.

29. (a) If $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2+(x+1)^2 y_1=2$.

OR

(b) If $x\sqrt{1+y}+y\sqrt{1+x}=0$, $-1<x<1$, $x\neq y$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

Sol. (a) Here $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

$\Rightarrow y=2\log\left(\frac{x+1}{\sqrt{x}}\right)=2\log(x+1)-\log x$

$\therefore \frac{dy}{dx}=\frac{2}{x+1}-\frac{1}{x}$

$\Rightarrow \frac{dy}{dx}=\frac{x-1}{x(x+1)}$

$\Rightarrow x\frac{dy}{dx}=\frac{x-1}{x+1}=1-\frac{2}{x+1}$

$\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0+\frac{2}{(x+1)^2}$

$\therefore x(x+1)^2\frac{d^2y}{dx^2}+(x+1)^2\frac{dy}{dx}=2$ i.e., $x(x+1)^2 y_2+(x+1)^2 y_1=2$.

OR

(b) We have $x\sqrt{1+y}+y\sqrt{1+x}=0$

$\Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$

On squaring both the sides, we get $x^2(1+y)=y^2(1+x)$

$\Rightarrow x^2+yx^2=y^2+xy^2$

$\Rightarrow x^2-y^2=xy^2-yx^2$

$\Rightarrow (x-y)(x+y)=xy(y-x)$

$\Rightarrow (x-y)(x+y)+xy(x-y)=0$

$\Rightarrow (x-y)(x+y+xy)=0$

$\therefore x+y+xy=0$ [$\because x\neq y \Rightarrow (x-y)\neq 0$

$\Rightarrow y(1+x)=-x$ i.e., $y=-\frac{x}{1+x}$

$$\Rightarrow y = -\frac{1+x-1}{1+x}$$

$$\Rightarrow y = -1+\frac{1}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = 0-\frac{1}{(1+x)^2}\times(0+1)$$

$$\therefore \frac{dy}{dx} = -\frac{1}{(1+x)^2}.$$

30. (a) A die with number 1 to 6 is biased such that $P(2)=\frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :

$A=\{(x, y): x+y=9\}$, $B=\{(x, y): x\neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

Sol. (a) Let E be the event of getting 2.

$$\therefore P(E)=\frac{3}{10},\ P(\bar{E})=1-P(E)=\frac{7}{10}.$$

Let X denotes the number of times number 2 appears on the dice, when it is thrown twice.

So, X can take values 0, 1, 2.

So, $P(X=0)=\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$, $P(X=1)={}^2C_1\left(\frac{3}{10}\times\frac{7}{10}\right)=\frac{42}{100}$, $P(X=2)=\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$.

Hence, the Mean $=\sum X.P(X)=0\times\frac{49}{100}+1\times\frac{42}{100}+2\times\frac{9}{100}=\frac{60}{100}=\frac{3}{5}=0.6$.

OR

(b) Here $A=\{(3, 6), (4, 5), (5, 4), (6, 3)\}$ and

$$B=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}.$$

Clearly, $A\cap B=\{(4, 5), (5, 4), (6, 3)\}$.

Now $P(A\cap B)=\frac{3}{36}=\frac{1}{12}$, $P(A)=\frac{4}{36}=\frac{1}{9}$, $P(B)=\frac{30}{36}=\frac{5}{6}$.

As $P(A)\times P(B)=\frac{1}{9}\times\frac{5}{6}=\frac{5}{54}\neq P(A\cap B)$ so, A and B are not independent events.

Further, $A\cap B\neq\phi$. So, A and B are not mutually exclusive.

31. Find $\int\frac{1}{x}\sqrt{\frac{x+a}{x-a}}dx$.

Sol. Let $I=\int\frac{1}{x}\sqrt{\frac{x+a}{x-a}}dx$

$$\Rightarrow I=\int\frac{1}{x}\sqrt{\frac{x+a}{x-a}\times\frac{x+a}{x+a}}dx$$

$$\Rightarrow I = \int \frac{1}{x} \times \frac{x+a}{\sqrt{x^2-a^2}}\,dx$$

$$\Rightarrow I = \int \frac{1}{x} \times \frac{x}{\sqrt{x^2-a^2}}\,dx + \int \frac{a}{x\sqrt{x^2-a^2}}\,dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{x^2-a^2}}\,dx + \int \frac{a}{x\sqrt{x^2-a^2}}\,dx$$

$$\Rightarrow I = \log\left|x+\sqrt{x^2-a^2}\right| + \sec^{-1}\left(\frac{x}{a}\right) + C$$

For second integral, put $x = a\sec\theta \Rightarrow \theta = \sec^{-1}\frac{x}{a}$. Also, $dx = a\sec\theta\tan\theta\,d\theta$

Then $\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \int \frac{a}{a\sec\theta\sqrt{a^2\sec^2\theta-a^2}} \times a\sec\theta\tan\theta\,d\theta$

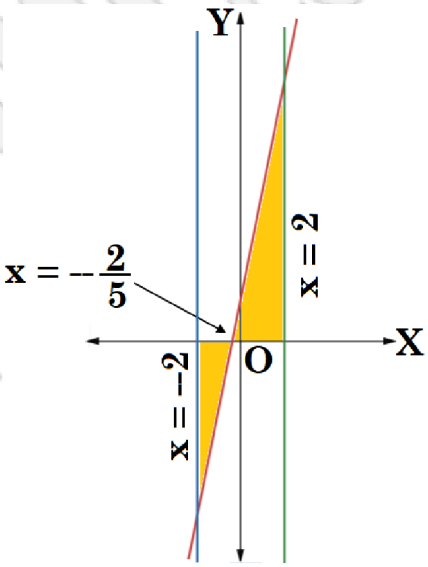
$$= \int \frac{a\tan\theta}{\sqrt{a^2(\sec^2\theta-1)}}\,d\theta = \int \frac{a\tan\theta}{a\tan\theta}\,d\theta = \int 1\,d\theta = \theta = \sec^{-1}\frac{x}{a}.$$

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. Using integration, find the area of the region bounded by the line $y = 5x+2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

Sol. Consider the diagram shown.



$$\text{Required area} = \left|\int_{-2}^{-\frac{2}{5}} (5x+2)\,dx\right| + \int_{-\frac{2}{5}}^{2} (5x+2)\,dx$$

$$= \left|\left[\frac{(5x+2)^2}{2\times5}\right]_{-2}^{-\frac{2}{5}}\right| + \left[\frac{(5x+2)^2}{2\times5}\right]_{-\frac{2}{5}}^{2}$$

$$= \left|\left[0 - \frac{64}{2\times5}\right]\right| + \left[\frac{144}{2\times5} - 0\right]$$

$$= \frac{64}{2\times5} + \frac{144}{2\times5}$$

$$= \frac{64}{2\times5} + \frac{144}{2\times5} = \frac{104}{5} \text{ Sq. units}.$$

33. Find $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$.

Sol. $$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{(x^2+1)+(x+2)-2}{(x+2)(x^2+1)}\,dx$$

$$\Rightarrow \quad = \int \frac{(x^2+1)}{(x+2)(x^2+1)}\,dx + \int \frac{(x+2)}{(x+2)(x^2+1)}\,dx - \int \frac{2}{(x+2)(x^2+1)}\,dx$$

$$\Rightarrow \quad = \int \frac{dx}{(x+2)} + \int \frac{dx}{(x^2+1)} - 2\int \left\{\frac{1}{5}\times\frac{1}{x+2} - \frac{1}{10}\times\frac{2x}{x^2+1} + \frac{2}{5}\times\frac{1}{x^2+1}\right\} dx$$

$$\Rightarrow \quad = \log|x+2| + \tan^{-1}x - 2\left\{\frac{1}{5}\times\log|x+2| - \frac{1}{10}\times\log|x^2+1| + \frac{2}{5}\times\tan^{-1}x\right\} + C$$

$$\Rightarrow \quad = \frac{3}{5}\log|x+2| + \frac{1}{5}\tan^{-1}x + \frac{1}{5}\times\log|x^2+1| + C.$$

34. (a) Find the shortest distance between the lines :

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of the line joining AA′. Find the foot of perpendicular from point A on the line l.

Sol. (a) For the given lines, $\vec{a}_1 = -\hat{i} + \hat{j} + 9\hat{k}$, $\vec{b}_1 = 2\hat{i} + \hat{j} - 3\hat{k}$; $\vec{a}_2 = 3\hat{i} - 15\hat{j} + 9\hat{k}$, $\vec{b}_2 = 2\hat{i} - 7\hat{j} + 5\hat{k}$.

So, $\vec{a}_2 - \vec{a}_1 = 4\hat{i} - 16\hat{j}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{i} - 16\hat{j} - 16\hat{k}$.

Therefore, $\text{S.D.} = \dfrac{\left|\vec{b}_1 \times \vec{b}_2 . (\vec{a}_2 - \vec{a}_1)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\Rightarrow \text{S.D.} = \frac{\left|(-16\hat{i} - 16\hat{j} - 16\hat{k}).(4\hat{i} - 16\hat{j})\right|}{16\sqrt{1^2+1^2+1^2}} = \frac{|-64+256|}{16\sqrt{3}} = \frac{192}{16\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ units}.$$

OR

(b) Rewriting the equation of line l in the Cartesian form, we get $\dfrac{x-4}{1} = \dfrac{y-2}{-1} = \dfrac{z-2}{-1} = \lambda$.

Given that A(2, 1, 2).

Assume that M be the foot of perpendicular on the line l drawn from A.

Any arbitrary point on the line is $M(\lambda+4, -\lambda+2, -\lambda+2)$.

Direction ratios of AM : $\lambda+2, -\lambda+1, -\lambda$.

Since AM is perpendicular to the given line l.

So by using $a_1a_2 + b_1b_2 + c_1c_2 = 0$ for $\perp$ lines, we get

$1(\lambda+2) + (-1)(-\lambda+1) + (-1)(-\lambda) = 0$

$\Rightarrow \lambda + 2 + \lambda - 1 + \lambda = 0$

$\Rightarrow \lambda = -\dfrac{1}{3}$.

$\therefore M\left(4-\frac{1}{3}, 2+\frac{1}{3}, 2+\frac{1}{3}\right) = M\left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right)$ is the **foot of perpendicular** of the point A in line.

Let image of point A in the line be A′(h, p, s).

Since M is the mid-point of AA′, so $M\left(\frac{2+h}{2}, \frac{1+p}{2}, \frac{2+s}{2}\right) = \left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right)$.

On comparing the respective coefficients, we obtain : $h = \frac{16}{3}$, $p = \frac{11}{3}$, $s = \frac{8}{3}$.

Therefore, the image of A in the line l is $A'\left(\frac{16}{3}, \frac{11}{3}, \frac{8}{3}\right)$.

The direction ratios of line AA' are $-\frac{1}{3}+2, \frac{1}{3}+1, \frac{1}{3}$ i.e., $\frac{5}{3}, \frac{4}{3}, \frac{1}{3}$ i.e., $5, 4, 1$.

Note that, since A, M and A' are on the same line. So, the d.r.'s of AA' will be same as that of the line AM.

Now the equation of line joining A and A' is $\frac{x-2}{5}=\frac{y-1}{4}=\frac{z-2}{1}$.

35. (a) Given $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find AB. Hence, solve the system of linear equations : $x-y+z=4,\ x-2y-2z=9,\ 2x+y+3z=1$.

OR

(b) If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find A^{-1}.

Hence, solve the system of linear equations : $x-2y=10,\ 2x-y-z=8,\ -2y+z=7$.

Sol. (a) $AB=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$

$\Rightarrow AB=8\,I$ ...(i)

Consider the given systems of equations : $x-y+z=4,\ x-2y-2z=9,\ 2x+y+3z=1$.

These equations can be expressed as : $BX=C$, where $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $C=\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$

Therefore, $X=B^{-1}C=\frac{1}{8}AC$ $\left[\text{By (i), } AB=8\,I\Rightarrow\left(\frac{1}{8}A\right)B=I\ \therefore B^{-1}=\frac{1}{8}A\right]$

So, $X=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$

$\therefore$ By equality of matrices : $x=3,\ y=-2,\ z=-1$.

OR

(b) For $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, $|A|=1(-1-2)-2(-2-0)+0=-3+4=1\neq 0$ $\therefore A^{-1}$ exists.

Adjoint of matrix A, $\text{adj.}A=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}^T=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$

Inverse of matrix A, $A^{-1} = \frac{\text{adj. A}}{|A|} = \frac{\text{adj. A}}{1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ ...(i)

The given system of equations can be represented as $A^TX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$.

Clearly, $X = (A^T)^{-1}B = (A^{-1})^T B$

By (i), $X = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}^T \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

By equality of matrices, we get $x = 0$, $y = -5$, $z = -3$.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y;\ x \in S,\ y \in J\}$.



Based on the above, answer the following.

(i) How many relations can be there from S to J?

(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective.

(iii) (a) How many one-one functions can be there from set S to set J?

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

Sol. **(i)** Here $n(S) = 4$, $n(J) = 3$ so, $n(S \times J) = 4 \times 3 = 12$.

Therefore, total number of relations from S to J are $2^{12} = 4096$.

(ii) Note that, $f(S_2) = J_2 = f(S_3)$. That is, S_2 and S_3 are both mapped to J_2.

Hence, f is not one-one.

Also, every element of J have at least one pre-image in S. Hence, f is onto.

Since f is onto but not one-one, so f is not bijective.

(iii) (a) As $n(S) = 4$, $n(J) = 3$ i.e., $n(S) > n(J)$.

So, number of one-one functions from set S to set J is 0 (zero).

OR

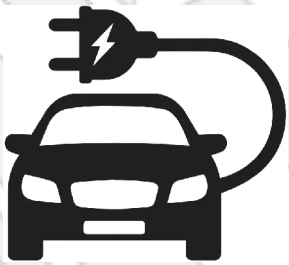
(iii) (b) For reflexivity, we must add the ordered pairs : $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$.

Since $(S_1, S_2) \in R_1$ and $(S_2, S_4) \in R_1$. So, we must **not** add the ordered pairs (S_2, S_1) and (S_4, S_2) in R_1, otherwise it will become symmetric.

Therefore, after adding **minimum number of ordered pairs** i.e., $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$ in R_1 so that it becomes reflexive but not symmetric, the new relation R_1 becomes $R_1 = \{(S_1, S_2), (S_2, S_4), (S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)\}$.

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively.

Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).



Based on the above, answer the following.

(i) (a) What is the probability that a randomly selected car is an electric car?

OR

(i) (b) What is the probability that a randomly selected car is a petrol car?

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet?

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi?

Sol. Let E_1, E_2 and E_3 be the events that the cars are manufactured by Amber, Bonzi and Comet respectively. Also let E be the event that the car is electric (or battery operated).

Clearly, $\overline{E}$ represents the event that the car is a petrol car.

Now $P(E_1) = 60\% = \frac{6}{10}$, $P(E_2) = 30\% = \frac{3}{10}$, $P(E_3) = 10\% = \frac{1}{10}$.

Also $P(E \mid E_1) = 20\%$, $P(E \mid E_2) = 10\%$, $P(E \mid E_3) = 5\%$.

That means, $P(\overline{E} \mid E_1) = 80\%$, $P(\overline{E} \mid E_2) = 90\%$, $P(\overline{E} \mid E_3) = 95\%$.

(i) (a) By law of total probability, $P(E) = P(E_1) \times P(E \mid E_1) + P(E_2) \times P(E \mid E_2) + P(E_3) \times P(E \mid E_3)$

$$\Rightarrow P(E) = \frac{6}{10} \times \frac{20}{100} + \frac{3}{10} \times \frac{10}{100} + \frac{1}{10} \times \frac{5}{100}$$

$$\Rightarrow P(E) = \frac{120 + 30 + 5}{1000} = \frac{155}{1000} = \frac{31}{200} = 0.155.$$

OR

(i) (b) By law of total probability, $P(\overline{E}) = P(E_1) \times P(\overline{E} \mid E_1) + P(E_2) \times P(\overline{E} \mid E_2) + P(E_3) \times P(\overline{E} \mid E_3)$

$$\Rightarrow P(\overline{E}) = \frac{6}{10} \times \frac{80}{100} + \frac{3}{10} \times \frac{90}{100} + \frac{1}{10} \times \frac{95}{100}$$

$$\Rightarrow P(\overline{E}) = \frac{480 + 270 + 95}{1000} = \frac{845}{1000} = \frac{169}{200} = 0.845.$$

Alternatively, we can find $P(E)$ and, then use $P(\bar{E}) = 1 - P(E)$.

(ii) By Bayes' theorem, $P(E_3 \mid E) = \dfrac{P(E \mid E_3) \times P(E_3)}{P(E)}$

$$\Rightarrow P(E_3 \mid E) = \frac{\frac{1}{10} \times \frac{5}{100}}{\frac{155}{1000}} = \frac{5}{155} = \frac{1}{31}.$$

(iii) $P(E_1 \mid E) + P(E_2 \mid E) = 1 - P(E_3 \mid E) = 1 - \dfrac{1}{31} = \dfrac{30}{31}$.

Alternatively, we can obtain $P(E_1 \mid E)$ and $P(E_2 \mid E)$ separately using Bayes' theorem and then add them.

38. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modeled by $f(x) = e^x \sin x$, where x is in metres.



Based on the above, answer the following.

(i) Find the intervals on which the $f(x)$ is increasing or decreasing $x \in [0, \pi]$.

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.

Sol. **(i)** For $f(x) = e^x \sin x$, $f'(x) = e^x \cos x + \sin x\, e^x = e^x (\cos x + \sin x)$

When $f'(x) = e^x (\cos x + \sin x) = 0$, then $(\cos x + \sin x) = 0$ [As $e^x \neq 0$

$\Rightarrow 1 + \tan x = 0 \quad \Rightarrow \tan x = -1$

$\therefore x = \dfrac{3\pi}{4} \in [0, \pi]$

Interval	Sign of $f'(x)$	Nature of $f(x)$
$\left[0, \frac{3\pi}{4}\right]$	Positive	Increasing
$\left[\frac{3\pi}{4}, \pi\right]$	Negative	Decreasing

(ii) Since $f'(x)$ changes its sign from Positive to Negative as x passes through the point $x = \dfrac{3\pi}{4}$.

Therefore, $x = \dfrac{3\pi}{4}$ is the point of local maximum.

☑ Q.P. Code - 65/2/2

01. The values of x for which the angle between the vectors $\vec{a} = 2x^2 \hat{i} + 4x \hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is

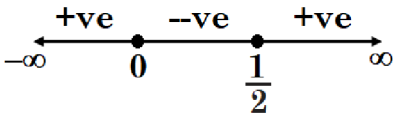
(a) 0 or $\frac{1}{2}$ (b) $x > \frac{1}{2}$ (c) $\left(0, \frac{1}{2}\right)$ (d) $\left[0, \frac{1}{2}\right]$

Sol. (c) For obtuse angle, $\cos\theta < 0$ i.e., $\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} < 0 \Rightarrow \vec{a}.\vec{b} < 0$

That is, $(2x^2\hat{i} + 4x\hat{j} + \hat{k}).(7\hat{i} - 2\hat{j} + x\hat{k}) < 0$

$\Rightarrow 14x^2 - 8x + x < 0 \quad \Rightarrow 14x^2 - 7x < 0$

$\Rightarrow 7x(2x-1) < 0$



$\therefore x \in \left(0, \frac{1}{2}\right)$.

03. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

(a) $\tan x + \cot x + C$ (b) $(\tan x + \cot x)^2 + C$

(c) $\tan x - \cot x + C$ (d) $(\tan x - \cot x)^2 + C$

Sol. (c) $\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{(\sin^2 x + \cos^2 x)\,dx}{\sin^2 x \cos^2 x} = \int (\sec^2 x + \operatorname{cosec}^2 x)\,dx = \tan x - \cot x + C$.

04. Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^{\alpha}$

(a) 0 (b) 1 (c) 2025 (d) $(2025)^3$

Sol. (b) Since det. value of a skew-symmetric matrix of odd order is always 0 (zero).

So, $\det(P) = \alpha = 0$. Therefore, $(2025)^{\alpha} = (2025)^0 = 1$.

05. The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is

(a) $-\frac{7\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{\pi}{10}$ (d) $\frac{3\pi}{5}$

Sol. (b) $\sin^{-1}\left(\cos\frac{43\pi}{5}\right) = \sin^{-1}\left(\cos\left\{9\pi - \frac{2\pi}{5}\right\}\right) = \sin^{-1}\left(-\cos\frac{2\pi}{5}\right)$

$= -\sin^{-1}\left(\cos\frac{2\pi}{5}\right) = -\sin^{-1}\left(\sin\left\{\frac{\pi}{2} - \frac{2\pi}{5}\right\}\right) = -\sin^{-1}\left(\sin\frac{\pi}{10}\right) = -\frac{\pi}{10}$.

11. In the following probability distribution, the value of p is

X	0	1	2	3
P(X)	p	p	0.3	2p

(a) $\frac{7}{40}$ (b) $\frac{1}{10}$ (c) $\frac{9}{35}$ (d) $\frac{1}{4}$

Sol. (a) Using $\sum P(X) = 1$, we get $p + p + 0.3 + 2p = 1$

$\Rightarrow 4p = 0.7 \qquad \therefore p = \frac{0.7}{4} = \frac{7}{40}$.

12. If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area (ΔPQR) is

(a) 2 Sq. units (b) 4 Sq. units (c) 6 Sq. units (d) 12 Sq. units

Sol. (c) area $(\Delta PQR) = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{\sqrt{16+64+64}}{2} = \frac{\sqrt{144}}{2} = \frac{12}{2} = 6$ Sq. units.

22. Evaluate $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx$, $p \in N$.

Sol. Let $f(x) = \frac{\sin 2px}{\sin x}$. Then $f(\pi - x) = \frac{\sin 2p(\pi - x)}{\sin(\pi - x)} = \frac{\sin(2p\pi - 2px)}{\sin x} = \frac{-\sin 2px}{\sin x} = -f(x)$.

By using $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)dx, \text{ if } f(2a-x) = f(x) \\ 0, \text{ if } f(2a-x) = -f(x) \end{cases}$, we get $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx = 0$, $p \in N$.

24. Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence find the values of λ and μ.

Sol. Using $\vec{r} = \lambda\vec{p} + \mu\vec{q}$, we get $\hat{i} + \hat{j} + 2\hat{k} = \lambda(2\hat{i} - 3\hat{j} - \hat{k}) + \mu(-3\hat{i} + 4\hat{j} + \hat{k})$.

Then $\hat{i} + \hat{j} + 2\hat{k} = \hat{i}(2\lambda - 3\mu) + \hat{j}(-3\lambda + 4\mu) + \hat{k}(-\lambda + \mu)$

On comparing the coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$, we get

$2\lambda - 3\mu = 1$, $-3\lambda + 4\mu = 1$, $-\lambda + \mu = 2$

Solving these equations, we get $\lambda = -7$, $\mu = -5$.

27. Prove that $f : N \to N$ defined as $f(x) = ax + b$, $(a, b \in N)$ is one-one but not onto.

Sol. Let $x_1, x_2 \in N$ such that $f(x_1) = f(x_2)$.

That is, $ax_1 + b = ax_2 + b \Rightarrow ax_1 = ax_2 \Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one.

Let $y = f(x) = ax + b$, where $y \in N$.

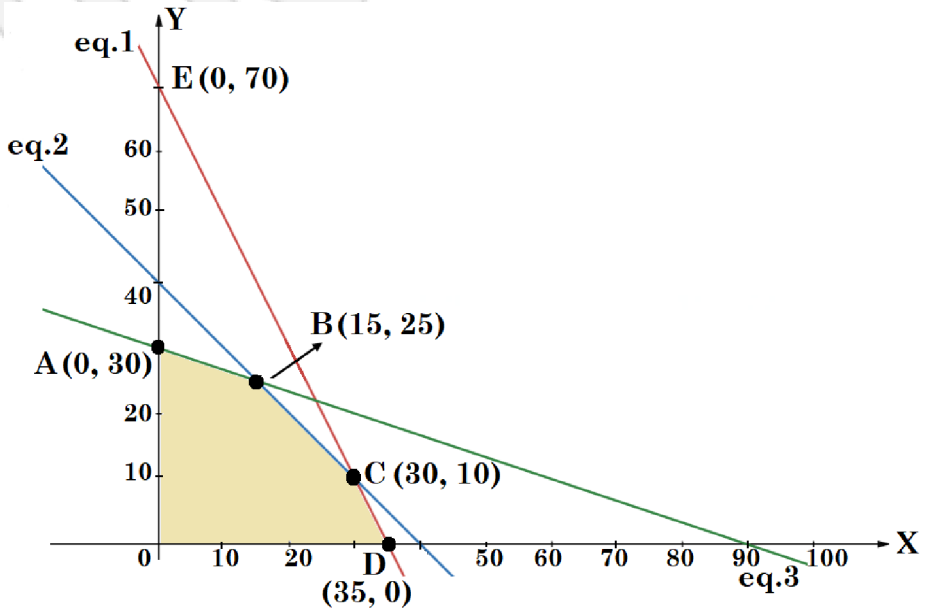
Note that, $y = ax + b \Rightarrow x = \frac{y - b}{a}$

Take $y = 1$, $b = 1$, $a \in N$. Then, $x = \frac{1-1}{a} = 0 \notin N$.

It is clear that there exists at least one natural number in the codomain (N) that is not mapped to any natural number x from the domain (N).

That means, Range ≠ Codomain . Hence, f is not onto.

28. The feasible region along with corner points for a linear programming problem is shown in the graph. Write all the constraints for the given linear programming problem.



Sol. Let's first of all write the equations.
As eq.1 passes through (35, 0) and (0, 70).
So, its equation is $\frac{y-0}{x-35}=\frac{70-0}{0-35} \Rightarrow 2x+y=70$.
As eq.2 passes through (40, 0) and (0, 40).
So, its equation is $\frac{y-0}{x-40}=\frac{40-0}{0-40} \Rightarrow x+y=40$
As eq.3 passes through (90, 0) and (0, 30).
So, its equation is $\frac{y-0}{x-90}=\frac{30-0}{0-90} \Rightarrow x+3y=90$
By taking in consideration the feasible region shown for LPP, we get the following constraints.
$2x+y\le 70$, $x+y\le 40$, $x+3y\le 90$, $x\ge 0$, $y\ge 0$.

31. f and g are continuous functions on interval $[a, b]$.
Given that $f(a-x)=f(x)$ and $g(x)+g(a-x)=a$, show that $\int_0^a f(x)g(x)\,dx=\frac{a}{2}\int_0^a f(x)\,dx$.

Sol. LHS : Let $I=\int_0^a f(x)g(x)\,dx$

$\Rightarrow I=\int_0^a f(a-x)g(a-x)\,dx$ [Using $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$\Rightarrow I=\int_0^a f(x)[a-g(x)]\,dx$ [Using $f(a-x)=f(x)$, $g(x)+g(a-x)=a$

$\Rightarrow I=\int_0^a [a\times f(x)-f(x)g(x)]\,dx$

$\Rightarrow I=a\times\int_0^a f(x)\,dx-\int_0^a f(x)g(x)\,dx$

$\Rightarrow I=a\times\int_0^a f(x)\,dx-I$

$\Rightarrow 2I=a\times\int_0^a f(x)\,dx \quad \therefore I=\frac{a}{2}\times\int_0^a f(x)\,dx$

Hence, $\int_0^a f(x)g(x)\,dx=\frac{a}{2}\int_0^a f(x)\,dx$.

33. Find $\int\frac{5x}{(x+1)(x^2+9)}dx$.

Sol. Let $I=\int\frac{5x}{(x+1)(x^2+9)}dx$

Consider $\frac{5x}{(x+1)(x^2+9)}=\frac{A}{x+1}+\frac{B(2x)+C}{x^2+9}$

$\Rightarrow 5x=A(x^2+9)+B\,2x\,(x+1)+C(x+1)$

On equating the coefficients of like terms on both the sides, we get : $A=-\frac{1}{2}$, $B=\frac{1}{4}$, $C=\frac{9}{2}$.

$$\text{So, } I = -\frac{1}{2}\int\frac{dx}{x+1} + \int\frac{\frac{1}{4}(2x)+\frac{9}{2}}{x^2+9}dx$$

$$\Rightarrow I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\int\frac{2x}{x^2+9}dx + \frac{9}{2}\int\frac{1}{x^2+9}dx \qquad \left[\because \int\frac{f'(x)}{f(x)}dx = \log|f(x)|+C\right.$$

$$\Rightarrow I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{9}{2}\times\frac{1}{3}\tan^{-1}\frac{x}{3} + C$$

$$\therefore I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{3}{2}\tan^{-1}\frac{x}{3} + C.$$

☑ Q.P. Code - 65/2/3

01. If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p}.\vec{q}$ is not possible?

(a) $-\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

Sol. (d) Since $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$, where θ is the angle between $\vec{p}$ and $\vec{q}$

$\Rightarrow \vec{p}.\vec{q} = 1\times1\cos\theta = \cos\theta$

As $-1 \le \cos\theta \le 1 \ \forall\ \theta \in \mathbb{R}$ so, $-1 \le \vec{p}.\vec{q} \le 1$.

Hence, $\vec{p}.\vec{q} \ne \sqrt{3}$.

03. If $\int_0^a x\,dx \le \frac{a}{2}+6$, then

(a) $-4 \le a \le 3$ (b) $a \ge 4,\ a \le -3$ (c) $-3 \le a \le 4$ (d) $-3 \le a \le 0$

Sol. (c) $\int_0^a x\,dx \le \frac{a}{2}+6$

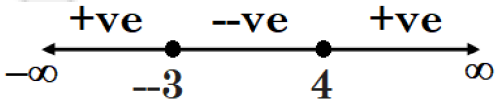
$$\Rightarrow \left[\frac{x^2}{2}\right]_0^a \le \frac{a}{2}+6$$

$$\Rightarrow \left[\frac{a^2}{2}-0\right] \le \frac{a}{2}+6$$

$\Rightarrow a^2 \le a+12 \qquad \Rightarrow a^2-a-12 \le 0$

$\Rightarrow (a-4)(a+3) \le 0$

$\therefore a \in [-3, 4]$.



04. If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a

(a) symmetric matrix (b) skew-symmetric matrix
(c) null matrix (d) unit matrix

Sol. (b) Let $P = (AB^T - BA^T)$

$$\Rightarrow P^T = (AB^T - BA^T)^T = (AB^T)^T - (BA^T)^T = (B^T)^T A^T - (A^T)^T B^T = BA^T - AB^T = -P$$

So, P is a skew-symmetric matrix.

05. The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$ is

(a) -1 (b) $-\frac{\sqrt{3}}{2}$ (c) 0 (d) 1

Sol. (a) $\cos\left(\frac{\pi}{6}+\cot^{-1}(-\sqrt{3})\right)=\cos\left(\frac{\pi}{6}+\frac{5\pi}{6}\right)=\cos\pi=-1$.

Note that, $\cot^{-1}(-\sqrt{3})=\pi-\cot^{-1}\sqrt{3}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$.

11. The probability distribution of a random variable X is given by

X	–4	–3	–2	–1	0
P(X)	0.1	0.2	0.3	0.2	0.2

Then E(X) of distribution is

(a) –1.8 (b) –1 (c) 1 (d) 1.8

Sol. (a) $E(X)=\sum X\,P(X)=(-4)(0.1)+(-3)(0.2)+(-2)(0.3)+(-1)(0.2)+(0)(0.2)$

$\Rightarrow E(X)=-0.4-0.6-0.6-0.2+0=-1.8$.

12. If projection of $\vec{a}=\alpha\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units, then α is

(a) –13 (b) –5 (c) 13 (d) 5

Sol. (d) According to question, $\vec{a}.\hat{b}=4$ i.e., $\frac{(\alpha\hat{i}+\hat{j}+4\hat{k}).(2\hat{i}+6\hat{j}+3\hat{k})}{\sqrt{2^2+6^2+3^2}}=4$

$\Rightarrow \frac{2\alpha+6+12}{\sqrt{49}}=4$

$\Rightarrow 2\alpha+18=4\times7=28 \qquad \Rightarrow 2\alpha=10 \qquad \therefore \alpha=5$.

22. Find $\int 2x^3 e^{x^2}\,dx$.

Sol. Put $e^{x^2}=t \Rightarrow 2x\,e^{x^2}dx=dt$. Also, $x^2=\log t$.

Now $\int 2x^3 e^{x^2}\,dx=\int x^2\times 2x\,e^{x^2}\,dx=\int \log t\,dt$ [Applying By Parts

$$=\log t\int 1\,dt-\int\left[\frac{d}{dt}(\log t)\int 1\,dt\right]dt$$

$$=\log t\times t-\int\left[\frac{1}{t}\times t\right]dt=\log t\times t-\int 1\,dt$$

$$=\log t\times t-t+C=(\log t-1)t+C$$

$$=(x^2-1)e^{x^2}+C.$$

24. If $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$, then evaluate $|\vec{a}+2\vec{b}|$.

Sol. Consider $|\vec{a}+2\vec{b}|^2=(\vec{a}+2\vec{b}).(\vec{a}+2\vec{b})$

$\Rightarrow |\vec{a}+2\vec{b}|^2=|\vec{a}|^2+|2\vec{b}|^2+2\vec{a}.2\vec{b}=|\vec{a}|^2+4|\vec{b}|^2+4\vec{a}.\vec{b}$

$\Rightarrow |\vec{a}+2\vec{b}|^2=2^2+4\times3^2+4\times4=4+36+16=56$

$\therefore |\vec{a}+2\vec{b}|=\sqrt{56}=2\sqrt{14}$.

27. Let R be a relation on set of real numbers $\mathbb{R}$ defined as $\{(x, y): x-y+\sqrt{3}$ is an irrational number, $x, y\in\mathbb{R}\}$. Verify R for reflexivity, symmetry and transitivity.

Sol. Note that, $x-x+\sqrt{3}=\sqrt{3}$ is an irrational number, for all $x\in\mathbb{R}$.
That is, $(x, x)\in R\ \forall\ x\in\mathbb{R}$. So, R is reflexive.

Take $x=\sqrt{3}$, $y=1$.

Then $\sqrt{3}-1+\sqrt{3}=2\sqrt{3}-1$ is an irrational number. But $1-\sqrt{3}+\sqrt{3}=1$ is not irrational.

That is, $(x, y) \in R$ but $(y, x) \notin R \ \forall \ x, y \in \mathbb{R}$. So, R is not symmetric.

Take $x = 1, y = \sqrt{2}, z = \sqrt{3}$.

Then $1 - \sqrt{2} + \sqrt{3}$ is irrational and $\sqrt{2} - \sqrt{3} + \sqrt{3} = \sqrt{2}$ is also irrational.

But $1 - \sqrt{3} + \sqrt{3} = 1$ is not irrational.

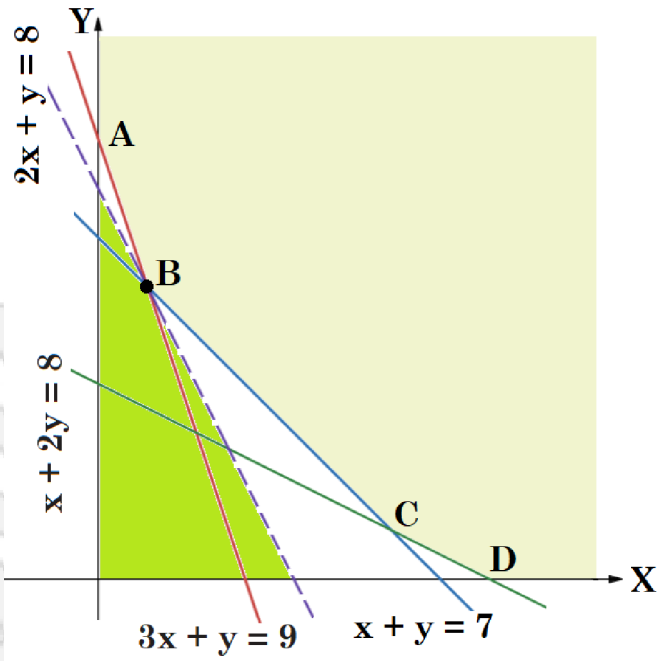
That is, $(x, y) \in R$ and $(y, z) \in R$ but $(x, z) \notin R \ \forall \ x, y, z \in \mathbb{R}$. So, R is not transitive.

28. Solve the following linear programming problem graphically.

Minimize $Z = 2x + y$

Subject to the constraints $3x + y \geq 9, \ x + y \geq 7, \ x + 2y \geq 8, \ x \geq 0, \ y \geq 0$.

Sol. Consider the graph shown below.



The corner points of feasible region are $A(0, 9)$, $B(1, 6)$, $C(6, 1)$ and $D(8, 0)$.

Corner Points	Value of Z
$A(0, 9)$	$Z_A = 2 \times 0 + 9 = 9$
$B(1, 6)$	$Z_B = 2 \times 1 + 6 = 8 \leftarrow$ Min.
$C(6, 1)$	$Z_C = 2 \times 6 + 1 = 13$
$D(8, 0)$	$Z_D = 2 \times 8 + 0 = 16$

Since the feasible region is unbounded so, $Z = 8$ may or may not be the minimum value of Z.

To check, let $2x + y < 8$.

Since there is no point common between the feasible region and $2x + y < 8$.

Hence, $Z = 8$ is the minimum value of Z and it is obtained at $(1, 6)$.

31. If $\int_a^b x^3 \, dx = 0$ and $\int_a^b x^2 \, dx = \frac{2}{3}$, then find the values of a and b.

Sol. $\int_a^b x^3 \, dx = 0 \quad \Rightarrow \left[\frac{x^4}{4}\right]_a^b = 0 \quad \Rightarrow \frac{b^4}{4} - \frac{a^4}{4} = 0 \quad \Rightarrow b^4 = a^4 \quad \Rightarrow b = \pm a \ \ldots(i)$

Also $\int_a^b x^2 \, dx = \frac{2}{3} \quad \Rightarrow \left[\frac{x^3}{3}\right]_a^b = \frac{2}{3} \quad \Rightarrow \frac{b^3}{3} - \frac{a^3}{3} = \frac{2}{3}$

$\Rightarrow b^3 - a^3 = 2$

By (i), when $b = a \ : \ a^3 - a^3 = 2 \quad \Rightarrow 0 = 2$, which is absurd.

By (i), when $b = -a \ : \ (-a)^3 - a^3 = 2$

$\Rightarrow -a^3 - a^3 = 2$

$\Rightarrow -2a^3 = 2 \quad \Rightarrow a^3 = -1 \quad \therefore a = -1$, and $b = -a = -(-1) = 1$.

33. Find $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$.

Sol. Let $I = \int (\sqrt{\cot x} + \sqrt{\tan x}) \, dx$

$$\Rightarrow I=\int\left[\frac{\sqrt{\cos x}}{\sqrt{\sin x}}+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}\right]dx$$

$$\Rightarrow I=\int\frac{\cos x+\sin x}{\sqrt{\sin x\cos x}}dx \qquad \left[\begin{array}{l}\text{Put } \sin x-\cos x=t\Rightarrow(\cos x+\sin x)dx=dt\\ \text{Also } (\sin x-\cos x)^2=t^2\Rightarrow 1-2\sin x\cos x=t^2\end{array}\right.$$

$$\therefore I=\int\frac{\sqrt{2}\,dt}{\sqrt{1-t^2}}$$

$$\Rightarrow I=\sqrt{2}\sin^{-1}[\sin x-\cos x]+C\,.$$

Alternatively, let $I=\int(\sqrt{\cot x}+\sqrt{\tan x})\,dx$

$$\Rightarrow I=\int\left[\frac{\sqrt{\cos x}}{\sqrt{\sin x}}+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}\right]dx \qquad \Rightarrow I=\int\frac{\cos x+\sin x}{\sqrt{\sin x\cos x}}dx$$

$$\Rightarrow I=\int\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)}{\sqrt{\sin x\cos x}}dx$$

$$\Rightarrow I=\int\frac{\sqrt{2}\left(\sin\frac{\pi}{4}\cos x+\cos\frac{\pi}{4}\sin x\right)}{\sqrt{\frac{2\sin x\cos x}{2}}}dx$$

$$\Rightarrow I=2\int\frac{\sin\left(\frac{\pi}{4}+x\right)}{\sqrt{\sin 2x}}dx \qquad \left[\text{Put } \frac{\pi}{4}+x=t\Rightarrow dx=dt\right.$$

$$\therefore I=2\int\frac{\sin t}{\sqrt{\sin\left(-\frac{\pi}{2}+2t\right)}}dt$$

$$\Rightarrow I=2\int\frac{\sin t}{\sqrt{-\cos 2t}}dt$$

$$\Rightarrow I=2\int\frac{\sin t}{\sqrt{1-2\cos^2 t}}dt \qquad [\text{Put } \cos t=u\Rightarrow \sin t\,dt=-du$$

$$\therefore I=-2\int\frac{du}{\sqrt{1-2u^2}}$$

$$\Rightarrow I=-2\int\frac{du}{\sqrt{1-(\sqrt{2}u)^2}}$$

$$\Rightarrow I=-2\times\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}u}{1}\right)+C$$

$$\Rightarrow I=-\sqrt{2}\sin^{-1}\left(\sqrt{2}\cos\left(\frac{\pi}{4}+x\right)\right)+C$$

$$\Rightarrow I=-\sqrt{2}\sin^{-1}\left(\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)\right)+C$$

$$\Rightarrow I=-\sqrt{2}\sin^{-1}(\cos x-\sin x)+C \text{ or, } I=\sqrt{2}\sin^{-1}(\sin x-\cos x)+C\,.$$

Alternatively, let $I = \int (\sqrt{\cot x} + \sqrt{\tan x})\,dx$

$$\Rightarrow I = \int \left\{ \frac{1}{\sqrt{\tan x}} + \sqrt{\tan x} \right\} dx$$

$$\Rightarrow I = \int \left\{ \frac{1+\tan x}{\sqrt{\tan x}} \right\} dx \qquad \left[\text{Put } \tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt \Rightarrow dx = \frac{2t\,dt}{1+t^4} \right.$$

$$\therefore I = \int \left\{ \frac{1+t^2}{\sqrt{t^2}} \right\} \frac{2t\,dt}{1+t^4}$$

$$\Rightarrow I = 2\int \frac{1+t^2}{1+t^4}\,dt$$

$$\Rightarrow I = 2\int \frac{\frac{1}{t^2}+1}{\frac{1}{t^2}+t^2}\,dt$$

$$\text{Put } t - \frac{1}{t} = u \Rightarrow \left(1+\frac{1}{t^2}\right)dt = du \;\&\; \left(t-\frac{1}{t}\right)^2 = u^2 \Rightarrow t^2 + \frac{1}{t^2} = u^2 + 2$$

$$\therefore I = 2\int \frac{du}{u^2+2}$$

$$\Rightarrow I = 2 \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$\Rightarrow I = \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{t\sqrt{2}}\right) + C$$

$$\therefore I = \sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + C\,.$$

With regards,
O.P. GUPTA
Author & Math Mentor
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